Numerical Methods in Matrix Computations. *By Åke Björck.* Springer, 2015. $99.00. xvi+800pp., hardcover. ISBN: 978-3-319-05088-1.

Matrix computations, or *numerical linear algebra*, is a fundamental part of numerical analysis, scientific computing, and computational mathematics. As a consequence, many books are devoted to this discipline. One can find in the literature general books that cover the most important parts of the subject, as for instance [2, 3, 4, 16, 20], monographs on more specific topics, as for instance [1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19], and even books dealing entirely with particular classes of matrices [17, 18]. This list of references is just a sample; many others are available on the market. In addition, general books on numerical analysis and scientific computing very often contain long chapters on matrix computations, and there are many references covering different applications of matrix computations to other areas. Therefore, this discipline is very well represented in the literature and, furthermore, some of the references are recent, or have been recently updated, as can be checked by inspecting the dates of some of the references cited above. In this scenario, the key question when reviewing a new book on matrix computations is precisely "what is new" in the book? Does the new book make any important addition to the existing literature? I advance that Björck's book *Numerical Methods in Matrix Computations* (NMMC) does indeed.

NMMC clearly belongs to the set of general references on matrix computations mentioned in the previous paragraph, where we also find the famous treatise by Golub and Van Loan (GVL) [3, 4], widely accepted as the "bible of the field", that is, as the most influential and complete reference on this subject. However, there is also a broad agreement in the community that GVL's monumental volume is too encyclopedic and advanced to serve as a general textbook for graduate students, and I have the personal feeling that is mainly used by researchers from different areas.

A first important remark on NMMC is that it is almost as encyclopedic as GVL, since NMMC is 800 pages long, while GVL 3rd and 4th editions are 694 and 756 pages long, respectively (in the case of the 4th edition, the full bibliography has not been counted). This remark is not just a matter of length, it is also a matter of contents, because NMMC covers essentially all the topics included in GVL, and on certain topics NMMC includes even more material than GVL. For instance, Chapter 2 of NMMC on least squares problems contains a 25 pages long section on nonlinear least squares problems, a topic not considered in GVL, a number of structured least squares problems not included in GVL, and a more complete treatment than GVL of regularization techniques. A key difference between NMMC and GVL is that NMMC is much easier to read, and it is less advanced in my opinion. This is related to the greatest merit of NMMC and what makes it unique: it can be read "almost" without pain and, simultaneously, includes a huge number of interesting topics and recent results. I have enjoyed a lot and learnt many new (at least for me) results while reading NMMC every day during the last few months in the train, on my way from home to the University, and, also very important, I have done it in a relaxed way!

How did the author write such highly readable and, at the same time, encyclopedic book? He mentions in the preface that "To keep the book within reasonable bounds, complete proofs are not given for all theorems". This is true, but it is also true that if not all, at least a fair number of details are very often provided, which combined with many insightful comments and intuitions, allow the reader to get a clear understanding, even in those parts where complete proofs are not given. All these comments naturally lead to the conclusion that NMMC can be used as a very adequate textbook on matrix computations for first year graduate students from a variety of disciplines. This is indeed the case, but, in my opinion, the guidance of the instructor will be crucial for guiding the students towards the most important results in matrix computations, among the many topics presented in the book.

Not surprisingly, the considerable extension of NMMC implies that it covers topics that are not covered, or are not covered with the same depth, in other excellent general references on matrix computations that are very adequate as graduate textbooks, as the much more concise Demmel's [2] or Trefethen and Bau's [16] books, or the highly readable book by Watkins [20]. For instance, NMMC pays considerable attention to sparse and structured linear systems (including Toeplitz, Hankel, Vandermonde, semiseparable linear systems, etc) and least squares problems (around 100 pages in total are devoted to these structured topics in sections 1.5, 1.7, 1.8, and 2.5), to special (weighted, with different constraints, generalized, total, etc) and nonlinear least squares problems (in sections 2.7 and 2.8), to tensor computations (in section 2.5.2), to functions of matrices (in the 23 pages long section 3.8), and to iterative methods for least squares problems (33 pages in section 4.5).

Another remarkable feature of NMMC that I appreciate very much is the significant attention paid to the historical evolution of matrix computations, which is explicitly and implicitly observed throughout the exposition. This can be explicitly observed, for instance, in the biographical footnotes on mathematicians who have made significant contributions to matrix computations that can be found in the text. They are very informative, are included in the right places, and are, simultaneously, brief enough not to disturb the natural flow of the text. The list of biographical notes can be found on page 788, and just a quick glance at it should convince the reader that it contains a carefully selected nonstandard list of researchers (including Byers, Dantzig, Francis, Golub, Kublanovskaya, Hestenes, Stiefel, Wilkinson, Young, etc, among many others), on whom it is worthwhile to get additional information. More importantly, NMMC includes very interesting historical comments, together with the corresponding references, on how and why some algorithms have arisen and evolved. These comments are often very nicely intermingled with the text and the mathematical developments, and supply additional insightful information on certain algorithms that is not found in other books. The historical remarks are not necessarily related to "old results", since they may refer to recent algorithms, and reveal that the book is written by someone who lived, and is still living, the development of matrix computations from the front lines. This is also reflected in the broad bibliography collected in NMMC, which includes both classical and very recent references, and also in the excellent and attractive way in which the references are cited.

The book is organized into four very long chapters: direct methods for linear systems, linear least squares problems, matrix eigenvalue problems, and iterative methods. Although they are called "chapters" by the author, these chapters are really different "parts" of the book since some of them are longer than 200 pages. This extreme chapter length causes some topics to remain a bit hidden on a first glance at the table of contents. For instance, the *LU* factorization is in subsection 1.2.2, the conjugate-gradient method in subsection 4.2.3, and the Arnoldi method in subsection 4.3.1, at the same level as other much less important topics. However, this is a second order issue and a matter of taste, since all the subjects contained in the book can be easily found.

NMMC contains, of course, all expected classical topics and results on matrix computations, but it also contains very recent material such as, for instance, a very readable section on tensor computations hidden in subsection 2.5.2, and several nonstandard topics on least squares problems. It is very hard to find relevant topics that are not covered in the book. Perhaps two somewhat striking omissions, from my perspective, are the solution of large-scale Sylvester matrix equations, a topic that has received much attention in the last decade, and the very brief treatment given to polynomial eigenvalue problems, another topic that has received much attention in recent years. But this is again a matter of personal taste, since it is not possible to cover all interesting topics in only one book.

In summary, this is a very well-written excellent book, which I strongly recommend to anybody interested in matrix computations and in computational mathematics in general. From my point of view, the two most distinctive features of NMMC that make it unique are: (1) it covers many topics, and all of them at a very accessible level, and (2) it is written with a penetrating historical style. These features imply that to read this book will be a pleasure, both for interested beginners and for experts in the field. As a consequence, NMMC can be very useful for a wide audience, ranging from graduate students to specialists in matrix computations, as well as researchers in other areas that use matrix computations.

References

[1] Å. Björck. *Numerical Methods for Least Squares Problems.* SIAM, Philadelphia, PA, 1996.

[2] J.W. Demmel. *Applied Numerical Linear Algebra.* SIAM, Philadelphia, PA, 1997.

[3] G.H. Golub and C.F. Van Loan. *Matrix Computations, 3rd edn.* Johns Hopkins University Press, Baltimore, MD, 1996.

[4] G.H. Golub and C.F. Van Loan. *Matrix Computations, 4th edn.* Johns Hopkins University Press, Baltimore, MD, 2013.

[5] A. Greenbaum. *Iterative Methods for Solving Linear Systems.* SIAM, Philadelphia, PA, 1997.

[6] N.J. Higham. *Accuracy and Stability of Numerical Algorithms, 2nd edn.*. SIAM, Philadelphia, PA, 2002.

[7] N.J. Higham. *Functions of Matrices. Theory and Computation.* SIAM, Philadelphia, PA, 2008.

[8] I.C.F. Ipsen. *Numerical Matrix Analysis. Linear Systems and Least Squares.* SIAM, Philadelphia, PA, 2009.

[9] J. Liesen and Z. Strakoš. *Krylov Subspace Methods. Principles and Analysis.* Oxford University Press, Oxford, 2013.

[10] M.A. Olshanskii and E.E. Tyrtyshnikov. *Iterative Methods for Linear Systems: Theory and Applications.* SIAM, Philadelphia, PA, 2014.

[11] B.N. Parlett. *The Symmetric Eigenvalue Problem.* SIAM, Philadelphia, PA, 1998 (republished ammended version of original published by Prentice-Hall, Englewood Cliffs, NJ, 1980).

[12] Y. Saad. *Iterative Methods for Sparse Linear Systems, 2nd edn.* SIAM, Philadelphia, PA, 2003.

[13] Y. Saad. *Numerical Methods for Large Eigenvalue Problems.* SIAM, Philadelphia, PA, 2011 (revised edition).

[14] G.W. Stewart. *Matrix Algorithms. Vol. I. Basic Decompositions.* SIAM, Philadelphia, PA, 1998.

[15] G.W. Stewart. *Matrix Algorithms. Vol. II. Eigensystems.* SIAM, Philadelphia, PA, 2001.

[16] L.N. Trefethen and D. Bau III. *Numerical Linear Algebra.* SIAM, Philadelphia, PA, 1997.

[17] R. Vandebril, M. Van Barel, and N. Mastronardi. *Matrix Computations and Semiseparable Matrices. Vol. 1. Linear Systems.* Johns Hopkins University Press, Baltimore, MD, 2008.

[18] R. Vandebril, M. Van Barel, and N. Mastronardi. *Matrix Computations and Semiseparable Matrices. Vol. II. Eigenvalue and Singular Value Methods.* Johns Hopkins University Press, Baltimore, MD, 2008.

[19] D.S. Watkins. *The Matrix Eigenvalue Problem. GR and Krylov Subspace Methods*. SIAM, Philadelphia, PA, 2007.

[20] D.S. Watkins. *Fundamentals of Matrix Computations, 3rd edn.* John Wiley and Sons, Hoboken, NJ, 2010.

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