Physics Department
Electricity and Magnetism Laboratory

INTERFERENCE AND DIFFRACTION

1. Goal

1) To study the diffraction pattern of a single slit,
2) to measure the diameter of a red blood cell,
3) to determine the wavelength of the light emitted by a helium-neon laser using a diffraction grating,
4) to study the interference between the light beams emerging from two closely-spaced slits,
5) to study light dispersion.

2. Overview

Light is an electromagnetic wave, and under the proper circumstances, it exhibits wave phenomena, such as constructive and destructive interference. The wavelength of visible light ranges from about 400-750 nm, and this wavelength $\lambda$ sets the scale for the appearance of wave-like effects. For instance, if a broad beam of light partly passes through a wide slit (i.e. a slit which is very large compared to $\lambda$), then the wave effects are negligible, the light acts like a ray, and the slit casts a geometrical shadow. However, if the slit is small enough (i.e. around the same size as $\lambda$ or smaller), then the wave properties of light become apparent and a diffraction pattern is projected.

![Figure 1a. Slit large compared to $\lambda$.](image1a.png)  ![Figure 1b. Slit small compared to $\lambda$.](image1b.png)

Now consider the light from two coherent light sources a distance $d$ apart. Coherent sources emit light waves that are in phase, or in sync. If we think of light like a water wave, we can imagine that coherent sources emit an identical succession of wave crests and troughs, with both emitting crests at the same time. One way to create such coherent sources is to illuminate a pair of narrow slits with a distant light source.

![Figure 2. Points A and B act like coherent sources.](image2.png)
Interference from two slits: Consider the light rays from the two coherent point sources made from slits a distance \(d\) apart (see fig. 3). We assume that the sources are emitting monochromatic (single wavelength) light of wavelength \(\lambda\). The rays are emitted in all forward directions, but let’s concentrate on the rays that are emitted in a direction \(\theta\) toward a distant screen (\(\theta\) measured from the normal to the screen, diagram below). One of these rays has further to travel to reach the screen, and the path difference is given by \(d \sin \theta\). When this path difference is exactly one wavelength \(\lambda\) (or any integer number of wavelengths) the interference will be constructive, while when the path difference is \(\lambda/2, 3\lambda/2, 5\lambda/2...\) the interference will be destructive.

A complete analysis yields a pattern of intensity vs. angle that looks like:

![Intensity pattern diagram]

\[
\begin{align*}
\text{Bright:} & \quad d \sin \theta = m\lambda \\
\text{Dark:} & \quad d \sin \theta = (m + \frac{1}{2})\lambda.
\end{align*}
\]

Geometric simplification: If \(\theta\) is small, then \(\sin \theta \approx \theta\) (in radians), and maxima occur on the screen at \(\theta = m \frac{\lambda}{d}\); minima occur at \(\theta = (m + \frac{1}{2}) \frac{\lambda}{d}\).

As shown below, the angle \(\theta\) (measured from the center of the screen) is related to the distance \(x\) measured on the screen by \(\tan \theta = x/L\), where \(L\) is the distance from the screen to the source of light (the aperture).

![Geometric simplification diagram]

If the angle \(\theta\) is small (less than a few degrees), then to an excellent approximation, \(\sin(\theta) \approx \tan(\theta) \approx \theta\) (in radians) so the locations of the interference maxima are given by

\[
\frac{x}{L} = m \frac{\lambda}{d}
\]
Single slit diffraction: The uniform 2-slit interference pattern shown above is seldom observed in practice, because real slits always have finite width (not an infinitesimal width). We now ask: what is the intensity pattern from a single slit of finite width D? Huygens’ Principle states that the light coming from an aperture is the same as the light that would come from a collection of coherent point sources filling the space of the aperture. It’s like we constructed the large slit out of a whole set of small slits, all adjacent to each other. To see what pattern the entire array produces, consider first just two of these imaginary sources: one at the edge of the slit and one in the center. These two sources are separated by a distance D/2.

The path difference for the rays from these two sources, going to the screen at an angle \( \theta \), is \( \frac{D}{2} \sin \theta \), and these rays will interfere destructively if \( \frac{D}{2} \sin \theta = \frac{\lambda}{2} \). But the same can be said for every pair of sources separated by D/2. Consequently, the rays from all the sources filling the aperture cancel in pairs, producing zero intensity on the screen when \( \frac{D}{2} \sin \theta = \frac{\lambda}{2} \) or, if \( \theta \) is small,

\[
\theta = \frac{\lambda}{D}
\]

(First minimum in single slit pattern.)

The complete intensity pattern, called a diffraction pattern, looks like

The single slit diffraction pattern has minima at

\[
\theta = \pm n \frac{\lambda}{D} \quad n = 1, 2, 3 \ldots
\]

(minima of single slit pattern.)

So the separation of minima is \( \lambda/D \), except for the first minima on either side of the central maximum, which are separated by \( 2\lambda/D \). If \( x \) is the distance on the screen between minima, then \( \theta = x/L = \lambda/D \).

Circular hole diffraction: For a circular hole, interference occurs between all rays from the whole area producing a series of concentric fringes. The exact solution in this case is difficult but turns out to be of the same form as for a slit (\( d \cdot \sin \theta = m \lambda \)) except that \( m \) is noninteger (i.e. \( m = 1.22, 2.23, 3.24, 4.24, 5.24 \ldots \)) and \( d \) is the diameter of the hole. The interference fringes produced by diffraction of laser light by a random array of these holes is shown in figure (the inset show the array).
Babinet's principle, which applies to any point outside the area illuminated by the undiffracted beam, states that the illumination is unaltered if the transparent parts of an aperture become opaque and the opaque parts transparent. Thus the same pattern of fringes will be produced by a circular hole in an opaque screen and by a circular disk of the same diameter as the hole. The effect of a not too dense and random array of holes is to enhance the intensity of the diffraction pattern due to a single cell. A blood smear on a microscope slide is an array of this type where particles are red blood cells. Diffraction is used to measure their diameters.

If the angular deviation is small \( \sin \theta \approx \theta \approx S_m/L \), where \( S_m \) is the radius of the \( m \)th dark fringe and \( L \) is the distance between the diffracting particles and the screen.

**Combine interference (2-slit) with diffraction (finite-width slit):** When the aperture consists of two finite slits, each of width \( D \), separated by a distance \( d \), then the intensity pattern is a combination of both the single-slit pattern and the double slit pattern: the amplitude of the two slit interference pattern is modulated by a single slit diffraction pattern:

In this full pattern, the finely spaced interference maxima are spaced \( \Delta \theta = \lambda/d \) apart, while the more widely spaced minima of the single-slit diffraction pattern are separated by \( \Delta \theta = \lambda/D \) or \( 2\lambda/D \).

**Diffraction grating:** The diffraction grating is an optical component used to spatially separate polychromatic light (white light) into its constituent optical frequencies. Diffraction gratings are used in such diverse fields as spectroscopy, colorimetry, metrology and laser optics. A large number of parallel, closely spaced slits constitutes a diffraction grating, characterized by the spacing \( d \). \( G = 1/d \) is the slit frequency or slit density, more commonly called "lines per millimeter".

The condition for maximum intensity is the same as that for the double slit:
\[ d \sin \theta = m\lambda \quad m = 0, 1, 2, 3 \ldots \]

However, angular separation of the maxima is generally much greater because the slit spacing is so small for a diffraction grating. The intensity maximum is very sharp and narrow, providing the high resolution for spectroscopic applications. The peak intensities are also much higher for the grating than for the double slit.

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**Dispersion:** The primary purpose of a diffraction grating is to disperse light spatially by wavelength. A beam of white light incident on a grating will be separated into its component wavelengths upon diffraction from the grating, with each wavelength diffracted along a different direction. Dispersion is a measure of the separation (either angular or spatial) between diffracted light of different wavelengths.

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### 3. Learn more...

- **SERWAY, RA & JEWETT, JW.** “Physics” Volume 1. 3rd edition Ed Thomson 2003 Chapters 15,16


4. **Equipment.**

1. Optical table  
2. Laser He-Ne (\(\lambda=632.8\) nm)  
3. Mirror  
4. Single-slit plate  
5. A blood smear on a microscope slide  
6. Double-slit plate  
7. Diffraction grating plate  
8. White light torch  

5. **Experimental procedure.**

5.1 **Diffraction pattern from single slit.**

The light source will be the He-Ne laser which produces a monochromatic beam with a wavelength of \(\lambda = 632.8\) nm and a beam diameter of about 1 mm. The power output of our lasers is about 1 mW, a small amount, but still enough to damage your retina if you look directly into the beam.

**NEVER LOOK INTO A LASER BEAM.**

1. Mount the laser so that the beam reflects on the mirror and deflects towards the laboratory wall.  
2. Mount the single slit near the mirror on the optical table.  
3. Choose a slit opening.  
4. Tape a piece of paper to the screen (laboratory wall). You should use this paper to mark the various diffraction and interference patterns that you observe on the screen.  
5. Adjust the direction of the mirror, and the position of the slit to give the clearest possible diffraction pattern on the screen.  
6. Measure the distance \(L\).  
7. Record the slit width \(a\).  
8. Mark on the paper the locations of each of the maxima, making sure to accurately record their positions and widths.  
9. Remove the paper from the screen.  
10. Measure the width of the central maximum (the distance between the first minima on either side of the maximum) as marked on the paper. You and your partner should do this twice each.  
11. Measure the distance between two neighboring dark fringes. In order to increase the accuracy of your measurement, select a group of five neighboring secondary maxima, measure the distance between the minima on either side, and divide it by five. (Can you explain why this procedure improves the accuracy of your measurement?) Again,
you and your partner should each do this twice. (Note: Make sure that the central minimum is not part of your group of five.)

12. Determine the best possible value for the separation of two dark fringes.

13. Compare the width of the central maximum to the average separation of two dark fringes. Is the width of the central maximum twice the separation of two dark fringes, as theory predicts?

14. Remount the paper on the screen.

15. Change the slit opening and repeat steps 4–12.

QUESTIONS

5.1.1 How does the observed single slit pattern change as you vary the width of the slit?

5.1.2 For both cases: Obtain experimentally the width of the slit (with the corresponding errors) and compare with the manufacturer’s value.

5.1.3 Which measurement is more accurate? Why?

5.2 Diffraction from a red blood cell

1. Set up the blood smear on a microscope slide such that the laser light through it.
2. Measure the distance $L$.
3. Measure the diameter of the first black circle to 5 different directions.

QUESTIONS

5.2.1 Obtain experimentally the diameter of a red blood cell with the corresponding error.

5.3 Double Slit Interference and Diffraction patterns

1. Set up the double slit at a distance from the mirror such that both slits are illuminated by the laser light. Select one pair of slits
2. Record both the slit widths and the distance between the two slits.
3. Measure five groups of 10 bright fringes to obtain a good average value for the distance between two dark fringes in the pattern.
4. Repeat steps 1–4 using a different double slit.

QUESTIONS

5.3.1 Note that the fringe separation depends only on $d$, the separation of the two slits. The intensities in the pattern, however, vary with the width of the individual slits. This is because the single-slit diffraction pattern serves as an intensity envelope for the double-slit interference pattern. Compare the diffraction patterns of different double slits to observe that the above rules are in fact correct.

5.3.2 Cover one of the two slits and see how the pattern on the screen changes.
5.3.3 What happens to the double-slit pattern a) as the distance between the slits is increased; b) as the width of each individual slit is increased?

5.3.4 Explain the contributions of both single-slit diffraction and double-slit interference to the double-slit pattern. How should the width of the single slits be chosen to perturb the double-slit pattern as little as possible?

5.3.5 For both cases: Obtain experimentally the separation of the slits (with the corresponding errors) and compare with the manufacturer’s value.

5.4 Diffraction Grating

1. Set up the diffraction grating plate and select one of them.
2. Record the line density of the selected grating.
3. Measure the distance between the orders of diffraction and calculate the diffraction angles.
4. Repeat steps 1 – 4 using a different diffraction grating.

QUESTIONS

5.4.1 For both gratings: Calculate experimentally the wavelength of the He-Ne laser (with the corresponding errors) and compare with its real value

5.5 Dispersion of visible light

1. Use the torch as light source and select the diffraction grating that resolves better the visible spectrum.
2. Record the line density of the selected grating.
3. Illuminate the diffraction grating with the torch.
4. Measure the distance between maxima for two different colors of the rainbow (blue, red).

QUESTIONS

5.5.1 Explain your selection of the diffraction grating. Which one is it? Why?

5.5.2 Make a sketch of the observed dispersion of light and explain in term of diffraction.

5.5.3 Calculate the wavelength of blue and red (with the corresponding errors). Compare with their theoretical values and with the wavelength of the He-Ne laser.